An APL game for the electrons

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Abstract

Electrons are physical entities the behaviour of which can be directly modelled, using, as a mathematical tool of research, a powerful programming language. This paper proposes a short introduction to some vectorial aspects of modulo 2 integer algebra that fit the most simple aspects of the properties of the populations in general and of electrons in particular.

Postulatum : All electrons know APL; they like playing with the function which reflects their properties : Then, what are these properties and, as a corollary, WHAT IS such a function ?

Even in 1994, we do not know much about the electron : it is so small ! The hereabove-emitted postulate should not sound more strange and bizarre than the one of quantum mechanics (a quasi-religion for over 60 years of modern physics) stating that electrons have a null radius r (which makes the absolute potential become infinite on the particle, then creates some difficulties when equations have to be integrated); however, other effects (e.g. the Compton effect [MacG]) have led some physicists to derive several (not zero) values, between 1E¯16 and 1E¯13 m(eter); much more precise are the values for *e*, the charge (always negative) and *m* the mass : the concept of mass contradicts the null value for the radius; hence the hypothesis that one electron occupies a volume (a *quantum* box) such that no other electron may squat the same box. Computed by physicists, the electron's lifetime is, by far, much longer than the estimated age of the Universe, so that, when an electron exists as a granule of matter, it can be assumed as a permanent entity.

Electrostatic phenomena were known in Antiquity; word "electron" comes from the Greek word, ελεκτρον, for "amber". The absolute property of electrons - which they share with other still more mysterious entities such as "magnetic masses", "spins", or, in everyday life, "sexes", can be written in natural language :

Entities with the same "sign" repel each other, while entities with opposed "signs" attract each other.

Now, physicists do consider charges and masses in electronic units rather than in *coulombs or grams,* because the electron cannot (for the moment ...) be thought of as an assemblage of smaller entities; and formulas already simplify in the absence of constants which were formerly expressed in function of macroscopic arbitrary units. Similarly, energies are widely measured in *eV* (electron-volts). So, one electron is coded 1 i.e. as ONE elementary mass together with ONE elementary charge; correlatively, we may take Ø for the "NO electron", i.e. for an empty quantum box with the same size as the elementary section of space, ("finitely" small), occupied by ONE electron; this model does not require a precise knowledge of the actual size of the quantum box: it may also fit other entities than electrons.

Schematically, we can use a white quad or Ø and a black quad or 1: □ ■ in order to visualise as well :

a) the empty state and the full state of the quantum box, respectively,

b) the action of not-modifying and the action of modifying these states, respectively.

Note. The action of not-modifying is equivalent to a no-action.

The elementary law

We shall try to show that the elementary law of electrostatics (also the one of magnetism and of gravitation), could now state that ONE quantum box cannot and may not contain more than ONE entity; for electrons, the "usual law" is usually expressed by the same rule as the one of multiplication, that all girls and boys learn by heart, at school.

+ times + is +, + times - is -, - times + is - while - times - is + (the most surprising part of the law, difficult to be admitted, when presented in this counter-intuitive manner to young children).

We shall not discuss here the case of anti-matter (for which + corresponds to the "positron", the anti-electron), which has nothing to do in everyday electronic interactions as they happen to occur in thunderstorms, general electronics (then computing) or in chemistry, then in biology : the various types of chemical bonds correspond, without exception, to properties of "regular" electrons. So, what does the "+" sign (or, rather, the symbolic grapheme "+") mean, in the sign rule, when this latter is applied to electrons ?

Symbol "-" happens to fit the fact that the elementary charge of the electron is indeed negative; (in reality, this is a pure coincidence, due to the discovery that electrons were found to swim upstream - like salmon females in the Columbia river - when a continuous current flows through an electric circuit from the conventional + pole to the conventional - pole of a battery). Symbol "+" does not refer to any existing positive charge, but subtly indicates a deficiency in negative charge, a hole between charges (or masses), i.e. an empty quantum box, indeed.

In the expression of the sign rule, one may use word "by" instead of "times"; then, if one substitutes 1 to "-" which refers to a full quantum box, and Ø to "+", as a renormalisation of the notation (and of the origin), in the four cases at the same time, what APL symbol will replace word "times" (or "by") so that the law of electrostatics is magically transformed into an APL expression that will hold for all 4 possibilities ?

Electrons know that there is NO issue other than "≠" (this refines the initial postulate that all electrons know APL; in fact, they have attended one of the first courses only).

The isomorphous algebras

The "Exclusive Or" function of logic also corresponds to "Plus modulo 2" or "Minus modulo 2" of Modulo 2 integer algebra, if 0 corresponds to "nothing" and 1 to "something" (and even to "everything which is not nothing" since this algebra only considers two values). The duality of both algebras (binary & integer modulo 2, the latter being named in mathematics the algebra of **Z/2Z**) allows, especially in APL (this is more difficult in FORTRAN or PASCAL), to express quantitative properties - because 0 and 1 ARE precise values in **Z/2Z** - now using a logical function, moreover on arrays. In addition, if necessary, functions may combine with what the APL standard names "operators" and mathematics "functionals" i.e. special functions acting on functions.

As expressed for the electrons, (α≠ω in α−ω notation), the sign rule is NOT complete, because, in physics, except in the cases of fusion (impossible with electrons) or of annihilation (impossible in the normal - e.g. chemical - case of "regular" matter) two interacting bodies, so two scalar arguments of the APL function, should produce two results; think of billiards (a classical example of classical mechanics) : after interaction of two balls - a collision -, two balls still roll on the table. Physical interactions between α and ω would be better expressed, then modelled, at least for 1-electrons (full quantum boxes) and "no electrons" (empty quantum boxes), if the result of the function contained the final state of the couple formed by both entities.

Like Baron Munchausen on his cannon-ball, let us jump - by thought - on the ⍺ quantum box : this hypotheses corresponds, mathematically, to set the origin on ⍺. When ⍺ is 0 (an empty quantum box), the resuft of ⍺≠⍵ is the same as the initial value of ⍵ ; in this operation, (the *no*-operation, or no-action, corresponding to monadic "plus" when complex algebra is not implemented in APL), the final couple (⍺, ⍵) is the same as the initial couple.

When (⍺ is 1 (a full quantum box), the result of ⍺≠⍵ is the initial value of ⍵, logically negated (~⍵ in APL), that will REPLACE the original value of ⍵; entity ⍺, the active one, has not been modified; entity ⍵, the patient, the passive one, has been modified :

The initial couple (⍺, ⍵) has become (⍺, ~⍵) when ⍺ is a full (then active) quantum box.

APL is the unique notation in mathematics, which allows to express not only electrostatics, but the heart of electro-dynamic interactions IN ALL CASES, with a simple mathematical formula :

The final state of a couple is the resuit of ≠\ applied to the initial state.

Application to "chemical computing"

If electrons appreciate APL, the reason lies in the preceding sentence; they will be able to exhibit their properties in some molecular structures which will allow ≠\ to become their Rule of the Game; such a case is met in the alternating double and single bonds of conjugated systems, e.g. in the the *all-trans-* retinal, our eyes' (rods & cones) logical unit i.e. computer heart. Moreover, electrons may not play any other rule, otherwise, the qualitative - now quantised - law of electrostatics would suffer exceptions; as far as we know, it never does. The consequences of such an hypothesis are exposed in the other paper : "The APL Theory of Human Vision".

In the example of billiards, the quantity of motion, or momentum *mv : mass* times *velocity,* is conserved; the ≠\ idiom conserves the meaning of information in sequences : entropy (disorder) will not grow, although the modulation will change all along the chain. No noise will appear. The average conformation (shape of the molecule) will correspond to the usually-drawn conventional static formula, with the

alternance =−=−=−= of rather-double & rather-single bonds, which one may write as : 1 0 1 0 1 0 1... in binary or **Z/2Z** notation.

When the leftmost double bond is taken as fixed, the chain will react (vibrate) as a string which is attached on the left side. (This happens in the retinal, the eye's computer, because the leftmost double bond belongs to a cycle of carbon atoms). When distance d_{C-C} of a bond coded 0 shortens, the bond becomes coded C=C

(i.e. 1), and conversely, so that 1 corresponds to "a higher number of filled quantum boxes" (macroscopically to a higher local electronic density). See as an example the coding of a sine curve in bit modulation, as given in the Appendix of "New Mathematics for the Computer" in which 1 0 1 0 1 0 etc... codes the *y* =0 signal as a sampled sine curve the oscillations of which are too small to become visible on a video screen.

In organic chemistry, two adjacent C=C bonds make an unstable system. If one among a pair of such bonds is fixed, the other one will become a C-C bond again : a leftmost fixed C=C may act on a rightmost either C=C or C-C bond, exactly as the left 1 of the pair either 1 1 or 1 0 acts on the rightmost scalar of the same pair by action of the ≠\ APL idiom. A single bond C-C as the leftmost scalar of a pair of adjacent bonds will have no action on the next rightmost bond : in chemistry, sequences such as C-C-C or C-C= C are indeed not unstable.

The isomorphism between, on one hand, what is possible for the behaviour of electrons as "co-operating" computing agents in these molecules - inter alia for vision processing - and, on the other hand, the APL vectorial binary algorithm ≠\ , arises, as soon as 0 and 1 are interpreted as THE constants of the **Z/2Z** algebra.

H.L. Resnikoff writes : "Mathematical processes that involve differentiation are unreliable unless the data is accurate, but integration processes are smoothing operators that spread the inaccuracies due to noise or to inadequacies of the measurement process over the collective ensemble of data." [Res]

This is true as far as conventional mathematics, with numbers and continuous functions, are used to model natural processes; but it does not hold anymore when natural processes are described using **Z/2Z** algebra (and modelled in APL using the adequate logical function), as the sign-rule proof shows it for the behaviour of the electrons, which ARE responsible for all the possible effects covered by chemistry, either in ions (cf. the Na/K transmission of information hypothesis in [Lana]) or in the covalent conjugated bonds of some organic compounds which play the role of information modulo 2-integrators (e.g. in retinal pigments), and, last but not least, in hydrogen bonds, in which our genetic patrimony may be encoded, between the base pairs of the DNA double-helix structure [Lanb].

≠\ the logical equivalent of +\ considered modulo 2, NEVER spreads inaccuracies, because the Cth iteration of B←≠\B with C←2⋆⌈2⍟1⌈⌈/,+/∨\B for any binary information B with finite length (in APL a vector if B is a sequence, but, more generally, also an array), always reproduces B exactly; *"no introduction of noise "* is a synonym expression of *"perfect computing ";* this property fully respects the 2nd principle of thermodynamics : "Life is Nature's solution to the problem of preserving information despite the second law of thermodynamics" cf. [Res, p. 74]. ≠\ never adds entropy to the data it works with; all other Boolean functions, scanned, i.e. propagated, (except =\ easily reducible to ≠\, since =/B produces the same result as ≠/B for 0≠2|⍴B, and the same result as ≠/1,B, for 1≠2|⍴B), damage information: they are by no means reversible, add entropy (noise) to the information they act upon; then, they may not be used for modelling living processes.

Comparison with a simulation by non-linear equations

One of the main objections about the ≠\ (APL-TOE) theory has come from the fact that modulo 2 integer algebra is linear, while most equations, used in many fields, (commonly taught and discussed in books) are non-linear.

Several answers can be brought to such an objection.

First, any modulation can be described (and is, very commonly) as a sequence of 0s and 1s; all programs, data, graphics, texts, in any computer, are sequences of 0s and 1s. Then, 0, as a number (even outside **Z/2Z**), raised to any positive power p is 0; similarly, 1, as a number (even outside **Z/2Z**), raised to any positive power p is 1. Hence the obliged linearity of ALL processes, as soon as they will be described in **Z/2Z**.

Second, all equations, which are nonlinear, describe processes as functions of historically-chosen parameters (in general, macroscopic ones, that were measurable or countable, e.g. pressure, temperature, resistance, density). These parameters lose their macroscopic meaning as soon as individuals are concerned: the same holds for all types of populations : the fact that every family has, as an average, 4.8 children in some countries, does indicates that food problems will soon appear, but brings no information on the way every individual family might solve or not solve the problem (starving, going West or reducing their offspring willingly).

The best example of a non-linear formula, which was forged around 1860 on an attraction-repulsion basis in order to explain population rates as a function of time, is Verhulst's equation; (one can also go back to the theory of the English priest Malthus, who preferred exponential laws) :

$X_{n+1} = 4\ \mu\ X_n\ (1 - X_n)$.

When the population increases more than food supply does, one can expect some "catastrophe" (following R. Thom), so that the population will decrease, until food becomes available again. Theoreticians have applied the same non-linear formula to explain cycles for epidemics, Wall Street quotations, earthquakes, solar bursts, or catches in fishing campaigns : it is a "chaotic formula" (indeed described in all books about fractals and chaos), as "long-range unpredictable" for some values of the μ constant, namely for μ=l, so with constant 4 alone in the formula. X is a population rate, which may vary between 0 and 1. Although, theoretically, 1 is a permitted value for X, the next generations would have a population of 0 people.

Constant 4 appeared in the "ad hoc" formula so that the result varies in the same "continuous" interval and may be re-injected into the equation for the next iteration.

In this equation, a "fixed point" for X=0.75 exists : X remains constant for all successive iterations. But what does this value mean, physically ? In fact, not much, at least before further investigation.

One starts understanding a little better if one admits that population growth, stability or decline CANNOT a) depend from a single variable, i.e. ONE parameter, b) is the result of the action or of the no-action of quantised individuals, necessarily acting as couples.

The most simple explanation consists in considering X itself as the averaged result of the status of two "hidden' quantised sub-variables, α and ω.

If both α and ω and have only two possible states, such as both extrema of X (either 0 or 1), only 4 cases are possible

α	ω	X_n (average) 0 .5 ×α+ω	$1-X_n$	X_{n+1}
0	0	0	1	0
0	1	0.5	0.5	1
1	0	0.5	0.5	1
1	1	1	1	0

The ecological non-linear formula has become... X_{n+1} ←α≠ω

Such an expression, again, expresses the sign rule, the Law of the electrons and of magnetic fields.

But one can go much further and show, even with a single "continuous" variable, that the non-linear equation can become linear in the general case, after some "renormalisation" :

Let us consider a new angular variable Y, now defined as `Y←○¯1○X*.5` in a way which is LESS *ad hoc* than the original choice of `X` now, `Y` is an angle, e.g. the argument of a complex number the module of which will always be 1, so that the corresponding number always lies on the trigonometric circle (note the ± symbol, which does not exist in APL, as well as the fact that two positive angles, as well as two negative angles may have the same sine, while the `¯1○ω APL` function has a unique result). Then, all the phenomena which seem to fit the Verhulst equation and which have, indeed, more or less periodic variations : DRY/WET years, biological or financial or astronomical cycles in general, will acquire a more realistic model, because, of course, `Y` is defined modulo `○2` .

Factor X_n becomes `(1○Y)*2`; factor 1-X_n becomes `(2○Y)*2` ipso facto (with some precautions for the intervals), so that the new resulting X_{n+1} becomes `(×/2,1 2○Y)*2.` Let us suppose we have lost the diskette which contained a powerful APL interpreter, or temporarily forgotten the password. Fortunately, APL is not a programming language only, but a tool of thought. So, before computing, we may reduce, first, the formula to `(1○2×Y)*2`; then, it appears..., when looking at the preceding text, that the new Y_{n+1} is simply TWICE the old one..., after complete elimination of the annoying trigonometry, which, always performed with floating-point arithmetic by computers, sometimes leads to disastrous truncations, which propagate errors in multi-iterated algorithms.

So, is the formula still non-linear ?

Now, one may answer an older pending question :

The "fixed point" `X=0.75` makes sense in physics, because it corresponds to `Y` as `F×(○2*0, +\N⍴ 1)÷3` with `N` any positive integer and `F` factor 180÷○1, then, for readers unfamiliar with the APL notation - to successive angles of 60 120 240 480 960... degrees, modulo 360 degrees of course, so that the angle soon oscillates between 120 and -120 degrees.

The associated "hidden" complex numbers which will correspond to a quasi-steady state of the phenomenon when the observable ratio is indeed X3÷4 , e.g. for an ideal control by the United Nations or by U.N.E.S.C.O. of the constancy of Earth population, are j and its complex conjugate j^2, the famous complex cubic roots of the unit 1. These constants exhibit most fantastic properties among numbers (together with 0 and 1, see hereabove) : they are, at the same time, the square, the inverse and the conjugate complex of one another. While it is impossible to store them exactly in the computer memory (even in extended APL implementations) using the traditional mathematical way (real part `¯0.5` twinned with and irrational imaginary part 0.75* .5) which spends 128 bits in IEEE precision, modulo 2 integer algebra, again, provides the necessary clue to overcome the difficulty, since it offers AN EXACT REPRESENTATION with ... 4 bits, as matrix G 2, the 2-geniton :

```
   ∣11∣     the square of which is  ∣10∣      the square
of which is    ∣11∣
   ∣10∣      (modulo 2)             ∣11∣          (modulo
```

2) |10|

All fractal Sierpinski matrices S obtained as `S←0≠2|V∘.!V←0,+\Nρ1` for any positive integer N, with dimension (size, shape) `2/N+1` will have this fantastic property; their modulo 2 square or modulo 2 matrix inverse is also `⌽⊖S` i.e. their 2nd-diagonal symmetric. Their cube is a unit matrix; The modulo 2 sum of the 3 matrices S, its square or inverse `S≠.∧S`, and of the conforming unit matrix is a null matrix (the sum of the n n^{th} roots of 1 is always 0). When `N+1` is a power of 2, such matrices, named genitons G, are symmetric matrices : every row (or column) is `≠\` applied to the preceding row or column. The following identity always holds: `G¯1⊖≠\G` as well as, by symmetry : `G-¯1⌽≠⍀G`.

The name "geniton" comes from the isomorphism (for `2 2 ρG`) with

X **X** the genetic sex matrix, also similar to the electronic spin matrix, combining
X **Y** both electron spin states, as proposed by Wolfgang Pauli already in the 1920's

⇑⇑ and as explained in previous papers.
⇑⇓

Did the successive powers of G2, the *j* matrix expressed modulo 2, inspire Fibonacci, at the end of the 12th century, long before Coulomb and Mendel laws (and sex chromosomes) were known, as well as complex algebra, logistic equations, Galois fields, etc... when he found that the reproduction of rabbit populations built his famous series ? See the story in [Brown] and try - in APL2, with regular arithmetic - expression : `+.×\Nρ⊂G2` with N any integer so that no limit or domain error occurs; look at all the numbers obtained, and compare each item of the result (integer matrices) with the arithmetic sums (each) of the two preceding ones.

A suggestion is to try in APL2 or TryAPL2 with : `N←12` and `⎕PW←255` then to write `2|` left of the expression, so as to try directly:

```
2|+.×\Nρ⊂2 2ρ 1 1 1 0
```

which produces the ternary TICK-TACK-ONE Fibonaccian pendulum of parities in **Z/2Z**, the Big Ben of the Universe and of Its genes at all levels.

Conclusion

Now, the rules of the game for the electrons, re-interpreted in APL, have become coherent at all scales, and are indeed connected to *j* the most important constant of physics : because *j* exists as matrices G, with any size (order, dimension, shape) from 2 to infinity, i.e. for two hidden variables as well as for as many as one will like.

j is the rotation symmetry-operator of the classical space-time cone, model of our Universe : for a growing universe the time-axis is the diagonal of a cube, each metric axis being one of the 3 axes of a trihedron Oxyz (the "bones" of the cone; think of a traditional lamp-shade); every new layer of events on every face Oxy , Oyz , Ozx , is built by successive rotations around the diagonal, *j* being THE rotation matrix that preserves the 3D "metric" symmetry. No three-dimensional universe could exist and evolve dynamically without *j* as its main transformer.

No correct general and accurate numeric representation of *j* can be thought of, except modulo 2. Conversely, if elementary interactions lead, starting from different considerations, to the SAME discovery of this universal matrix as THE operator, then, what such an operator builds must be a 3-dimensional fractal and Fibonaccian Universe just like ours.

In addition, the algebra of this model may make another quite important postulate vanish : time irreversibility becomes implicit, because, modulo 2, there is NO minus sign; (so, one cannot change anymore any sign in no equation or no Hamiltonian...; the *time-arrow* gets fixed... for ever - cf. [Lann]).

`2|-\ω` has the same effect as `2|+\ω` i.e the one of `≠\ω` : to create our future in a ONE-WAY stream, with no return ticket available either for us or for the electrons.

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